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Solving equations using logs

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We can use logarithms to solve equations where the unknown is in the power as in, for example, $4^x = 15$. Whilst logarithms to any base can be used, it is common practice to use base 10, as these are readily available on your calculator.

Examples

Example

Solve the equation $4^x = 15$.

Solution

We can solve this by taking logarithms of both sides. So,

$$\log 4^x = \log 15$$

Now using the laws of logarithms, and in particular $\log A^n = n \log A$, the left hand side can be re-written to give

$$x \log 4 = \log 15$$

This is more straightforward. The unknown is no longer in the power. Straightaway, dividing both sides by $\log 4$,

$$x = \frac{\log 15}{\log 4}$$

This value can be found from a calculator. Check that this equals 1.953 (to 3 decimal places).

Example

Solve the equation $6^x = 2^{x-3}$.

Solution

Take logarithms of both sides.

$$\log 6^x = \log 2^{x-3}$$

Now use the laws of logarithms.

$$x \log 6 = (x-3) \log 2$$

Notice now that the x we are trying to find is no longer in a power. Multiplying out the brackets

$$x \log 6 = x \log 2 - 3 \log 2$$

Rearrange this equation to get the two terms involving x on the right hand side:

$$3 \log 2 = x \log 2 - x \log 6$$

Factorise the right hand side by extracting the common factor of x.

$$\begin{aligned} 3\log 2 &= x(\log 2 - \log 6) \\ &= x\log\left(\frac{1}{3}\right) \end{aligned}$$

using the laws of logarithms. And finally $x = \dfrac{3\log 2}{\log\left(\frac{1}{3}\right)}$.

This value can be found from a calculator. Check that this equals -1.893 (to 3 decimal places).

Example

Solve the equation $\mathrm{e}^x = 17$.

Solution

We could proceed as in the examples above. However note that the logarithmic form of this expression is $\log_{\mathrm{e}} 17 = x$ from which, with the use of a calculator, we can obtain x directly as 2.833.

Example

Solve the equation $10^{2x-1} = 4$.

Solution

The logarithmic form of this equation is $\log_{10} 4 = 2x - 1$ from which

$$\begin{aligned} 2x &= 1 + \log_{10} 4 \\ x &= \frac{1 + \log_{10} 4}{2} \\ &= 0.801 \qquad (\text{ to 3 d.p.}) \end{aligned}$$

Example

Solve the equation $\log_2(4x+3) = 7$.

Solution

Writing the equation in the alternative form using powers we find $2^7 = 4x + 3$ from which
$x = \dfrac{2^7 - 3}{4} = 31.25$

Exercises

1. Solve (a) $6^x = 9$, (b) $4^{-x} = 2$, (c) $3^{x-2} = 1$, (d) $15^{2x+1} = 7$.
2. Solve the equation $\log(5x+2) = 3$.
3. Solve the equation $2^{1-x} = 5$.

Answers

1. (a) $x = \dfrac{\log 9}{\log 6}$, (b) $x = -\dfrac{\log 2}{\log 4} = -\dfrac{1}{2}$, (c) $x = 2$, (d) $x = \dfrac{1}{2}\left(\dfrac{\log 7}{\log 15} - 1\right)$.

2. $x = \dfrac{10^3 - 2}{5} = 199.6$. 3. $x = 1 - \log_2 5 = -1.322$ (3 d.p.).